MA0301 Exercise 8

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1

a) Since there is only one president, the possibilities is the sum of candidates

$$5 + 8 = 13$$

b) For every candidate from one party there is all the candidates of the other party to be compared to. Therefore the amount of possibilities is

$$5 \cdot 8 = 13$$

2

a) To end up with the amount of possibilities, we have to multiply the amounts of components together

$$4 \cdot 12 \cdot 3 \cdot 2 = 288$$

b) This reduces the amount of colors from 4 to 1. Therefore the amount of possibilities is

$$1 \cdot 4 \cdot 3 \cot 2 = 24$$

3

a) Let one bakery item be either pastry or muffins.

$$(8 + 6) \cdot (4 + 6 + 1 + 1) = 14 \cdot 12 = 168$$

b)

$$14 \cdot 4 \cdot 6 \cdot 6 = 2016$$

c)

$$8 \cdot 6 \cdot 6 \cdot 1 \cdot 2(14 \cdot 4) = 32256$$

$\boxed{4}$

$$_{8!}P_{8!} = \frac{8!}{(8-8)!} = 8! = 40320$$

$\boxed{5}$

a)

$$_{7}P_{2} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \cdot 6 = 42$$

b)

$$_{8}P_{4} = \frac{8!}{(8-4)!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$$

c)

$$_{10}P_{7} = \frac{10!}{(10-7)!} = 10 \cdot 9 \cdot \ldots \cdot 4 = 604800$$

d)

$$_{12}P_{3} = \frac{12!}{(12-3)!} = 12 \cdot 11 \cdot 10 = 1320$$

$\boxed{6}$

a)

$$_{10}C_{4} = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!} = \frac{10 \cdot \cancelto{3}{9} \cdot \cancelto{1}{8} \cdot 7}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}} = 10 \cdot 3 \cdot 7 = 210$$

b)

$$_{12}C_{7} = \frac{12!}{7!5!} = \frac{\cancelto{1}{12} \cdot 11 \cdot \cancelto{1}{10} \cdot 9 \cdot 8}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5}} = 11 \cdot 9 \cdot 8 = 792$$

c)

$$_{14}C_{12} = \frac{14!}{12!2!} = \frac{14 \cdot 13}{2} = 91$$

d)

$$_{15}C_{10} = \frac{15!}{10!5!} = \frac{\cancelto{1}{15} \cdot \cancelto{7}{14} \cdot 13 \cdot \cancelto{3}{12} \cdot 11}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5}} = 7 \cdot 13 \cdot 3 \cdot 11 = 3003$$

$\boxed{7}$

a)

If there are no restrictions, the amount of arrangements will be the number of permutations of the books

$$_{7}P_{7} = 7!$$

In order for the languages to alternate, the 3 books has to be inbetween each of the 4 books

There are $_{3}P_{3}$ permutations of the C++ books. For each of those permutations, there are $_{4}P_{4}$ permutations of the Java books.

Hence the amount of ways to arrange the books will be

b)

$$_{3,3}P \cdot {}_{4}P_{4} = 3! \cdot 4! = 144$$

Because all the C++ books has to be together, we can think of them like one "block" in the permutations of 5 blocks where four of the blocks are Java books.

For each of those blocks, there is $_{3}P_{3}$ arrangements of the C++ books.

Hence the amount of ways to arrange the books will be

c)

$$_{3}P_{3} \cdot {}_{5}P_{5} = 3! \cdot 5! = 720$$

Here, there are just two blocks. Therefore there is only $_{2}P_{2}$ permutations of the blocks

For each of those permutations, there are $_{3}P_{3}$ ways to arrange the C++ books.

And for each way to arrange the C++ books, there are $_{4}P_{4}$ ways to arrange the java books.

Hence the amount of ways to arrange the books will be

d)

$$_{2}P_{2} \cdot {}_{3}P_{3} \cdot {}_{4}P_{4} = 288$$

$\boxed{8}$

a) Because we don't care if there's a different order that the people were selected, we have to use combinations.

No restrictions means every combination of 12 in 20 people.

$$_{20}C_{12} = 125970$$

b) For every combination of six women, we have every combination of six men.

$$_{10}C_{6} \cdot {}_{10}C_{6} = 44100$$

c) Here we sum together all the combinations where there is an even number of women for every corresponding combination of men.

$$\sum_{i=0}^{5} {}_{10}C_{2i} \cdot {}_{10}C_{2(6-i)} = 63090$$

d) In order for the selection to contain more women than men, the amount of women has to be 7 so that the amount of men is 5.

Therefore, we sum together all combination products from 7 to 10.

$$\sum_{i=7}^{10} {}_{10}C_{i} \cdot {}_{10}C_{12-i} = 40935$$

e) Sum of all combination products from 8 to 10.

$$\sum_{i=8}^{10} {}_{10}C_{i} \cdot {}_{10}C_{12-i} = 10695$$

$\boxed{9}$

In order to solve this task, we will sum together separate cases

Case i) The number only contains one distinct digit from $\{1, 3, 7, 8\}$

This would be all the permutations of the digits, that is

$${}_4P_4$$

Case ii) The number contains 2 of the digit 3 and two distinct digits from $\{1, 7, 8\}$

Here, we start with all the ways we can form a four digit number including two of the digit 3.

Since we don't care what order the 3s are in, we want the combinations (for example, $x33x$ and $x33x$ are the same)

Therefore the amount of ways we can write a four digit number including two of the digit 3 would be

$${}_4C_2$$

For each of those ways to write the number, there are all the permutations of the remaining digits ways to construct a number (here order does matter since the digits are distinct)

Hence, the amount of ways we can write this number would be

$${}_4C_2 \cdot {}_3P_3$$

Case iii) The number contains 2 of the digit 7 and two distinct digits from $\{1, 3, 8\}$

This is the same as Case ii, just with 7s instead of $3s$

Case iv) The number contains 2 of the digit 3 and 2 of the digit 7

In this case, the amount of combinations would be the same as the amount of the ways we can write a four digit number with two fixed numbers.

Hence, the amount of ways we can write this number would be

$$_4C_2$$

In conclusion, the total amount of distinct four digit integers we can make with the digits 1, 3, 3, 7, 7, 8 would be

$$_4P_4 + {}_4C_2 \cdot {}_3P_3 \cdot 2 + {}_4C_2 = 102$$

$\boxed{10}$

Here i wrote a program to calculate a modified pascal triangle and print the coefficients of a specific row

scripts/pascal.py

```python
from math import comb

def pascal(n, left, right):
  i = 1
  result = [[left, right]]

  while i < n:
    prev_row = result[-1]
    result.append(
      [left ** (i+1)]
    + [comb(i+1, j+1) * (left ** (len(prev_row)-j-1)) *
       (right ** (j+1)) for j in range(len(prev_row)-1)]
    + [right ** (i+1)])

    i += 1

  return result

def pretty_print_pascal(triangle):
  for i, row in enumerate(triangle):
    print(f'{i+1}.\t{row}')

def pretty_print_row(row):
```

```python
  for i, coefficient in enumerate(row):
    print(f'{coefficient}\tx^{len(row)-i-1}\ty^{i}')


if __name__ == "__main__":

  print('a. ')
  pretty_print_row(pascal(12, 1, 1)[-1])
  print()

  print('b. ')
  pretty_print_row(pascal(12, 1, 2)[-1])
  print()

  print('c. ')
  pretty_print_row(pascal(12, 2, -3)[-1])
```

Here are the results:

```
a.
1       x^12    y^0
12      x^11    y^1
66      x^10    y^2
220     x^9     y^3
495     x^8     y^4
792     x^7     y^5
924     x^6     y^6
792     x^5     y^7
495     x^4     y^8
220     x^3     y^9
66      x^2     y^10
12      x^1     y^11
1       x^0     y^12

b.
1       x^12    y^0
24      x^11    y^1
264     x^10    y^2
1760    x^9     y^3
7920    x^8     y^4
25344   x^7     y^5
59136   x^6     y^6
101376  x^5     y^7
126720  x^4     y^8
112640  x^3     y^9
67584   x^2     y^10
24576   x^1     y^11
4096    x^0     y^12
```

```
c.
4096    x^12    y^0
-73728  x^11    y^1
608256  x^10    y^2
-3041280        x^9     y^3
10264320        x^8     y^4
-24634368       x^7     y^5
43110144        x^6     y^6
-55427328       x^5     y^7
51963120        x^4     y^8
-34642080       x^3     y^9
15588936        x^2     y^10
-4251528        x^1     y^11
531441  x^0     y^12
```

Which means that the coefficient for x^9y^3 for each of the subexercises would be

a)

$$220$$

b)

$$17601$$

c)

$$-3041280$$