MA0301 Exercise 6

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1

a)

In order to show that this is a partial order, the relation has to be reflexive, antisymmetric and transitive

However, it is not antisymmetric because

$$4 - 2 \bmod 2 = 0 \land 2 - 4 \bmod 2 = 0$$

b)

In order to show that this is a partial order, the relation has to be reflexive, antisymmetric and transitive

However, it is not antisymmetric because

$$(1,2)R(1,3) \land (1,3)R(1,2) \land (1,2) \neq (1,3)$$

2

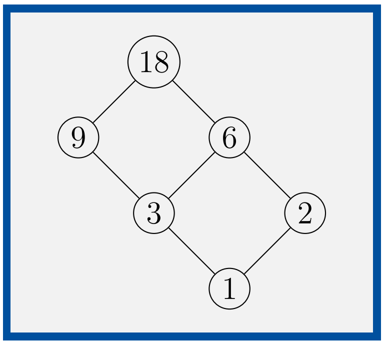

Figure 1: Hasse diagram of R

3

a)

In order to show that this is a partial order, the relation has to be reflexive, antisymmetric and transitive

Reflexive:

$$\begin{gathered}(a < a) \vee ((a = a) \wedge b \leqslant b)\\ F \vee (T \wedge T)\\ F \vee T\\ T\end{gathered}$$

Antisymmetric:

Case i)

$$a < c \Rightarrow a \neq c \wedge \neg(a < a)$$

Case ii)

$$\begin{gathered}(a, b) \neq (c, d) \wedge (a = c) \wedge (b \leqslant d) \Rightarrow b \neq d \Rightarrow b < d\\ \therefore (a = c) \wedge (b \leqslant d) \Rightarrow \neg(a < c) \wedge \neg(d \leqslant b)\end{gathered}$$

Transitive:

$$(a, b)R(c, d) \wedge (c, d)R(e, f) \Rightarrow (a, b)R(e, f)$$

This will be a proof by cases. In each case, I'm going to assume only one of the expressions in R turned out true, and show that it means that at least one of the expressions will be true as a result.

Case i and i)

$$(a < c) \wedge (c < e) \Rightarrow a < e$$

Case i and ii)

$$(a < c) \wedge (c = e \wedge d \leqslant f) \Rightarrow a < e$$

Case ii and i)

$$(a = c \wedge b \leqslant d) \wedge (c < e) \Rightarrow a < e$$

Case ii and ii)

$$(a = c \wedge b \leqslant d) \wedge (c = e \wedge d \leqslant f) \Rightarrow (a = f \wedge b \leqslant f)$$

b)

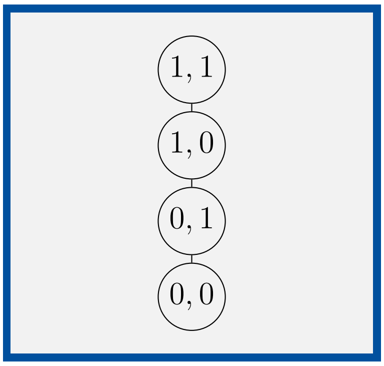

Figure 2: Hasse diagram of R

$(0, 0)$ is the only minimal element and $(1, 1)$ is the only maximal element in R.

c)

Since R only has one minimal and one maximal element, it is a total order.

4

a) This is a function because x can be expressed in terms of y
Range of $f(\mathbb{Z})$: $\{x \mid \pm\sqrt{x-7} \in \mathbb{Z}\}$

b) This is not a function because

$$x = (\pm y)^2$$

c) This is a function because x can be expressed in terms of y
Range of $f(\mathbb{R})$: $\mathbb{R}$

d) This is not a function because

$$x = \pm\sqrt{-y^2 + 1}$$

5

a)

$$f(x) = 2x - 3$$

One to one: ✓
Onto: ✗
Range of $f(\mathbb{Z})$: $\{x \mid x \bmod 2 = 1\}$

b)

$$f(x) = x^2$$

One to one: ✗
Onto: ✗
Range of $f(\mathbb{Z})$: $\{x \mid \sqrt{x} \in \mathbb{Z}\}$

c)

$$f(x) = x^3 + x$$

One to one: ✓
Onto: ✗
Range of $f(\mathbb{Z})$: $\{x \in f(\mathbb{Z})\}$ See message at ovsys

6

a)

$$f(x) = 2x - 3$$

One to one: ✓
Onto: ✓
Range of $f(\mathbb{R})$: $\mathbb{R}$

b)

$$f(x) = x^2$$

One to one: ✗
Onto: ✗
Range of $f(\mathbb{R})$: $\{x \mid x \geqslant 0\}$

c)

$$f(x) = x^3 + x$$

One to one: ✓
Onto: ✓
Range of $f(\mathbb{R})$: $\mathbb{R}$