MA0301 Exercise 3

Øystein Tveit

NTNU

1

$$
\begin{aligned}
&\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) && \\
&\neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q) && \text{Distributive law} \\
&\neg(\neg p \wedge T) \vee (p \wedge q) && \text{Complement law} \\
&\neg(\neg p) \vee (p \wedge q) && \text{Identity law} \\
&p \vee (p \wedge q) && \text{Double negation law} \\
&p && \text{Absortion law}
\end{aligned}
$$

2

$$
\begin{aligned}
&((p \wedge q) \vee (p \wedge \neg r) \vee \neg(\neg p \vee q)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \\
&((p \wedge q) \vee (p \wedge \neg r) \vee (\neg\neg p \wedge \neg q)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{De Morgans's law} \\
&((p \wedge q) \vee (p \wedge \neg r) \vee (p \wedge \neg q)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{Double negation law} \\
&((p \wedge (q \vee \neg q)) \vee (p \wedge \neg r)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{Distributive law} \\
&((p \wedge T) \vee (p \wedge \neg r)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{Complement law} \\
&((p) \vee (p \wedge \neg r)) \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{Identity law} \\
&p \vee ((r \vee s \vee \neg r) \wedge \neg q) && \text{Absortion law} \\
&p \vee ((\neg q \wedge r) \vee (\neg q \wedge s) \vee (\neg q \wedge \neg r)) && \text{Distributive law} \\
&p \vee (\neg q \wedge r) \vee (\neg q \wedge s) \vee (\neg q \wedge \neg r) && \text{Associative law}
\end{aligned}
$$

3

i)

a)

$$
\begin{gathered}
\{\{2,3,5\} \cup \{6,4\}\} \cap \{4,6,8\} \\
\{\{2,4,6\}\} \cap \{4,6,8\} \\
\emptyset
\end{gathered}
$$

b)

$$P(\{7,8,9\}) - P(\{7,9\})$$
$$\{\{7,8,9\},\{7,8\},\{8,9\},\{7,9\},\{7\},\{8\},\{9\},\emptyset\} - \{\{7,9\},\{7\},\{9\},\emptyset\}$$
$$\{\{7,8,9\},\{7,8\},\{8,9\},\{8\}\}$$

c)

$$P(\emptyset)$$
$$\{\emptyset\}$$

d)

$$\{1,3,5\} \times \{0\}$$
$$\{\langle 1,0\rangle,\langle 3,0\rangle,\langle 5,0\rangle\}$$

e)

$$\{2,4,6\} \times \emptyset$$
$$\emptyset$$

f)

$$P(\{0\}) \times P(\{1\})$$
$$\{\emptyset,\{0\}\} \times \{\emptyset,\{1\}\}$$
$$\{\langle\emptyset,\emptyset\rangle,\langle\emptyset,\{1\}\rangle,\langle\{0\},\emptyset\rangle,\langle\{0\},\{1\}\rangle\}$$

g)

$$P(P(\{2\}))$$
$$P(\{\emptyset,\{2\}\})$$
$$\{\{\{\emptyset\},\{2\}\},\{\{\emptyset\}\},\{\{2\}\},\emptyset\}$$

ii) Because the elements in a power set can be represented as a binary tree where every leaf node is a set that has the cardinality of 1, and that $\{\{x\} : x \in A\}$ would make up all the leaf nodes, we can reason that

$$|P(A) - \{\{x\} : x \in A\}| = \frac{n}{2}$$

$\boxed{4}$

a) $\emptyset = \{\emptyset\}$ is False because $|\emptyset| \neq |\{\emptyset\}|$

b) $\emptyset = \{0\}$ is False because $|\emptyset| \neq |\{0\}|$

c) $|\emptyset| = 0$ is True because $\emptyset$ has 0 elements

d) $P(\emptyset)$ is False because $P(\emptyset) = \{\{\emptyset\}\}$ has 1 element

e) $\emptyset = \{\}$ is True because the empty set is a subset of every possible set

f) $\emptyset = \{x \in \mathbb{N} : x \leq 0 and x > 0\}$ is False because $x \leq 0 \wedge x > 0 \equiv F$, which means there are no such elements, and thus the set is empty

$\boxed{5}$

a)

$$\begin{aligned}
&A \cap (A \cup B)\\
&\{x : x \in A \wedge x \in (A \cup B)\}\\
&\{x : x \in A \wedge (x \in A \vee x \in B)\}\\
&\{x : x \in A\}\\
&A
\end{aligned}$$

b)

$$\begin{aligned}
&A - (B \cap C)\\
&\{x : x \in A \wedge x \notin (B \cap C)\}\\
&\{x : x \in A \wedge (x \notin B \wedge x \notin C)\}\\
&\{x : (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C)\}\\
&\{x : x \in (A - B) \vee x \in (A - C)\}\\
&\{x : x \in (A - B) \cup (A - C)\}\\
&(A - B) \cup (A - C)
\end{aligned}$$

$\boxed{6}$

i)

$$
\begin{aligned}
&(A \cup B) \setminus (A \cap B)\\
&\{x : x \in (A \cup B) \setminus (A \cap B)\}\\
&\{x : x \in (A \cup B) \land x \notin (A \cap B)\}\\
&\{x : (x \in A \lor x \in B) \land (x \notin A \lor x \notin B)\}\\
&\{x : x \in A \land (x \notin A \lor x \notin B) \lor x \in B \land (x \notin A \lor x \notin B)\}\\
&\{x : ((x \in A \land x \notin A) \lor (x \in A \land x \notin B)) \lor ((x \in B \land x \notin A) \lor (x \in B \land x \notin B))\}\\
&\{x : (F \lor (x \in A \land x \notin B)) \lor ((x \in B \land x \notin A) \lor F)\}\\
&\{x : (x \in A \land x \notin B) \lor (x \in B \land x \notin A)\}\\
&\{x : x \in (A - B) \lor x \in (B - A)\}\\
&\{x : x \in (A - B) \cup (B - A)\}\\
&(A - B) \cup (B - A)
\end{aligned}
$$

ii) For this exercise, I counted the elements which was in either set but not both

$$A \Delta B = \{2, 4, 6, 7, 8\}$$

7

$$
\begin{aligned}
X &= \{\{1, 2, 3\}, \{2, 3\}, \{ef\}\} \cup \{\{e\}\}\\
&= \{\{1, 2, 3\}, \{2, 3\}, \{ef\}, \{e\}\}
\end{aligned}
$$

$$
\begin{aligned}
P(x) = \{ & \\
& \{\{1,2,3\},\{2,3\},\{ef\},\{e\}\}, \\
& \{\{1,2,3\},\{2,3\},\{ef\}\}, \\
& \{\{1,2,3\},\{2,3\},\{e\}\}, \\
& \{\{1,2,3\},\{ef\},\{e\}\}, \\
& \{\{2,3\},\{ef\},\{e\}\}, \\
& \{\{1,2,3\},\{2,3\}\}, \\
& \{\{1,2,3\},\{e\}\}, \\
& \{\{1,2,3\},\{ef\}\}, \\
& \{\{2,3\},\{ef\}\}, \\
& \{\{2,3\},\{e\}\}, \\
& \{\{ef\},\{e\}\}, \\
& \{\{e\}\}, \\
& \{\{ef\}\}, \\
& \{\{2,3\}\}, \\
& \{\{1,2,3\}\} \\
\}
\end{aligned}
$$

$$
\begin{aligned}
P(X \cap Y) &= P(\{\{1,2,3\},\{2,3\},\{ef\},\{e\}\} \cap \{\{1,2,3,e,f\}\}) \\
&= P(\emptyset) \\
&= \{\emptyset\}
\end{aligned}
$$

$\boxed{8}$

a) Here, the exercise says "[...] four sets A_1, A_2, A_3". I'm not sure if I'm supposed to do three or four, but I'll assume three sets A_1, A_2, A_3 was the intention.

$$
\begin{gathered}
A_1 \cap A_2 \cap A_3 \\
A_1 \cap A_2 \cap \overline{A_3} \\
A_1 \cap \overline{A_2} \cap A_3 \\
A_1 \cap \overline{A_2} \cap \overline{A_3} \\
\overline{A_1} \cap A_2 \cap A_3 \\
\overline{A_1} \cap A_2 \cap \overline{A_3} \\
\overline{A_1} \cap \overline{A_2} \cap A_3 \\
\overline{A_1} \cap \overline{A_2} \cap \overline{A_3}
\end{gathered}
$$

b) For each set, the amount of fundamental products is multiplied by 2. Therefore, the amount of fundamental sets of n sets is 2^n

$\boxed{9}$

$$
\begin{gathered}
A\overline{(B\overline{C})((A\overline{B})\overline{C})} \\
A(\overline{B}+\overline{\overline{C}})\overline{(A\overline{B}\,\overline{C})} \\
A(\overline{B}+C)\overline{(A\overline{B}\,\overline{C})} \\
A(\overline{B}+C)\overline{(A\overline{B}\,\overline{C})} \\
A(\overline{B}+C)(\overline{A}+\overline{\overline{B}}+\overline{\overline{C}}) \\
A(\overline{B}+C)(\overline{A}+B+C) \\
(A\overline{B}+AC)(\overline{A}+B+C) \\
A\overline{B}(\overline{A}+B+C)+AC(\overline{A}+B+C) \\
(A\overline{B}\,\overline{A}+A\overline{B}B+A\overline{B}C)+(AC\overline{A}+ACB+ACC) \\
(0+0+A\overline{B}C)+(0+ACB+AC) \\
A\overline{B}C+ACB+AC \\
ACB+AC \\
AC
\end{gathered}
$$

$\boxed{10}$ LHS

$$
\begin{gathered}
((A+B)+(A+C))\overline{((A+B)(A+C))}\,\overline{A} \\
(A+B+A+C)\overline{(A+B)(A+C)}\,\overline{A} \\
(A+B+C)(\overline{(A+B)}+\overline{(A+C)})\overline{A} \\
(A+B+C)(\overline{A}\,\overline{B}+\overline{A}\,\overline{C})\overline{A} \\
(\overline{A}A+\overline{A}B+\overline{A}C)(\overline{A}\,\overline{B}+\overline{A}\,\overline{C}) \\
(0+\overline{A}B+\overline{A}C)(\overline{A}\,\overline{B}+\overline{A}\,\overline{C}) \\
(\overline{A}B+\overline{A}C)(\overline{A}\,\overline{B}+\overline{A}\,\overline{C}) \\
\overline{A}B\overline{A}\,\overline{B}+\overline{A}B\overline{A}\,\overline{C}+\overline{A}C\overline{A}\,\overline{B}+\overline{A}C\overline{A}\,\overline{C} \\
0+\overline{A}B\overline{A}\,\overline{C}+\overline{A}C\overline{A}\,\overline{B}+0 \\
\overline{A}B\overline{C}+\overline{A}C\overline{B}
\end{gathered}
$$

RHS

$$(B+C)\overline{(BC)}\overline{A}$$
$$(B+C)(\overline{B}+\overline{C})\overline{A}$$
$$(\overline{A}B+\overline{A}C)(\overline{B}+\overline{C})$$
$$\overline{A}B\overline{B}+\overline{A}B\overline{C}+\overline{A}C\overline{B}+\overline{A}C\overline{C}$$
$$0+\overline{A}B\overline{C}+\overline{A}C\overline{B}+0$$
$$\overline{A}B\overline{C}+\overline{A}C\overline{B}$$

$LHS = RHS$