# MA0301 Exercise 2

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## 1

| $p$ | $q$ | $p \wedge q$ | $p \vee (p \wedge q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | F |
| F | F | F | F |

## 2

(DL1)

| $\alpha$ | $\beta$ | $\gamma$ | $(\beta \wedge \gamma)$ | $\alpha \vee (\beta \wedge \gamma)$ | $\alpha \vee \beta$ | $\alpha \vee \gamma$ | $(\alpha \vee \gamma) \wedge (\alpha \vee \gamma)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | F | T | F | F |
| F | F | T | F | F | F | T | F |
| F | F | F | F | F | F | F | F |

(DL2)

| $\alpha$ | $\beta$ | $\gamma$ | $(\beta \vee \gamma)$ | $\alpha \wedge (\beta \vee \gamma)$ | $\alpha \wedge \beta$ | $\alpha \wedge \gamma$ | $(\alpha \wedge \gamma) \vee (\alpha \wedge \gamma)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T |
| T | F | T | T | T | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

$\boxed{3}$

$$\begin{aligned} p \Rightarrow (q \vee r) &\equiv (p \wedge \neg q) \Rightarrow r \\ &\equiv \neg(p \wedge \neg q) \vee r \\ &\equiv (\neg p \vee \neg\neg q) \vee r \\ &\equiv (\neg p \vee q) \vee r \\ &\equiv \neg p \vee (q \vee r) \\ &\equiv p \Rightarrow (q \vee r) \end{aligned}$$

$\boxed{4}$

$$[(q \wedge p) \vee q] \wedge \neg(\neg q \vee p) \equiv q \wedge \neg p$$

$$\begin{aligned} q \wedge \neg p &\equiv [(q \wedge p) \vee q] \wedge \neg(\neg q \vee p) \\ &\equiv [q] \wedge (\neg\neg q \wedge \neg p) \\ &\equiv q \wedge (q \wedge \neg p) \\ &\equiv (q \wedge q) \wedge \neg p \\ &\equiv q \wedge \neg p \end{aligned}$$

$\boxed{5}$

**a)** $\forall S(x)[H(x)]$

**b)** $\exists S(x)[\neg H(x)]$

**c)** $\forall S(x)[\neg H(x)]$

**d)** $\forall \neg H(x) \exists S(x)$

$\boxed{6}$

**a)** The formula is true because of the case where $x < z < y$ which would mean that $x < y \wedge z < y \wedge x < z \wedge \neg(z < x)$

**b)** The formula is false because $p(z, y)$ and $\neg p(z, x)$ cannot be fulfilled at the same time. $z \geq 0 \wedge \neg(z \geq 0) \equiv F$

**e)** $\boxed{\text{see comment in ovsys}}$

$\boxed{7}$

$$\begin{aligned} \neg(\forall x[p(x) \wedge q(x)]) &\equiv \exists x[\neg(p(x) \wedge q(x))] \\ &\equiv \exists x[\neg p(x) \vee \neg q(x)] \end{aligned}$$

$\boxed{8}$

$$
\begin{aligned}
\neg(\exists x \forall y [p(y) \vee \neg q(x,y)]) &\equiv \forall x \neg(\forall y [p(y) \vee \neg q(x,y)]) \\
&\equiv \forall x \exists y \neg[p(y) \vee \neg q(x,y)] \\
&\equiv \forall x \exists y [\neg p(y) \wedge \neg\neg q(x,y)] \\
&\equiv \forall x \exists y [\neg p(y) \wedge q(x,y)]
\end{aligned}
$$