MA0301 Exercise 12
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$\boxed{1}$

$$r = \{a, b\}^* a \{a, b\}^* a \{a, b\}^* a \{a, b\}^*$$

$\boxed{2}$

a)

$$\{ab\}\{ab\}^*$$

b)

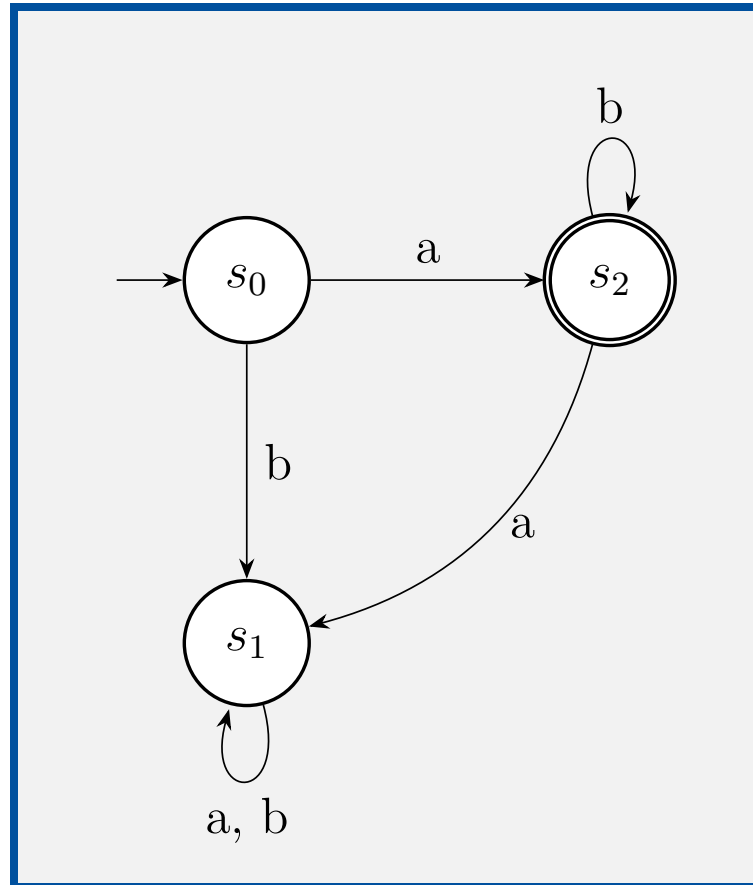$$a(a|\lambda)b(b|\lambda)$$

$\boxed{3}$

$$M = (Q, \Sigma, \delta, s, F)$$

$$Q = \{s_0, s_1, s_2\}$$
$$\Sigma = \{a, b\}$$
$$\delta = \begin{Bmatrix} s_0 \xrightarrow{b} s_1, \\ s_0 \xrightarrow{a} s_2, \\ s_1 \xrightarrow{a,b} s_1, \\ s_2 \xrightarrow{a} s_1, \\ s_2 \xrightarrow{b} s_2 \end{Bmatrix}$$
$$s = s_0$$
$$F = \{s_2\}$$

4

The words L can be described by the regular expression r where

$$r = a^*bb^*a\{a,b\}^*$$

5

The words in L can be described by the regular expression r where

$$r = (a^*b)^3\{(a^*b)^4\}$$

6

a)

$$s_0 \xrightarrow{a,0} s_0$$
$$s_0 \xrightarrow{a,0} s_0$$
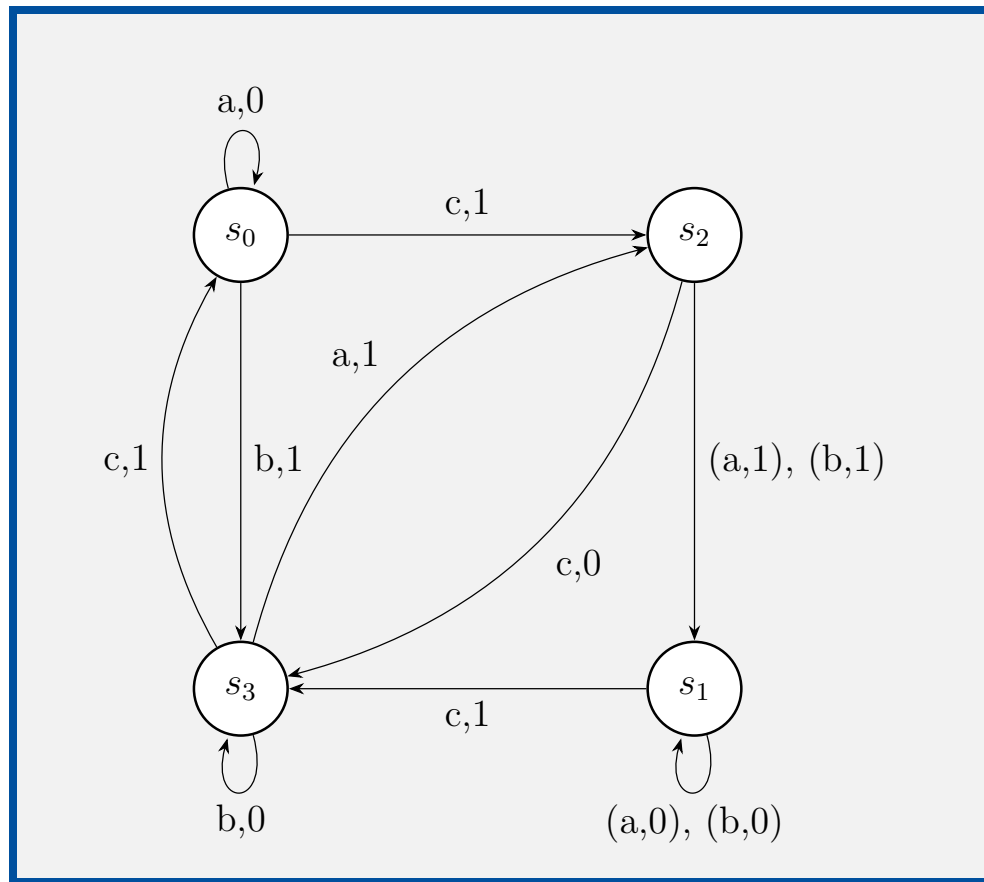$$s_0 \xrightarrow{b,1} s_3$$
$$s_3 \xrightarrow{b,0} s_3$$
$$s_3 \xrightarrow{c,1} s_0$$
$$s_0 \xrightarrow{c,1} s_2$$

The output would be 001011

b)

$\boxed{7}$

a)   Suppose we have $a \in A, b \in B$

$$
\begin{aligned}
AB^* &= \{a, ab, ab^2, ab^3, \ldots\} \\
&= \{a\} \cup \{ab, ab^2, ab^3, \ldots\} \\
&= A \cup \{ab, ab^2, ab^3, \ldots\} \\
&\Rightarrow A \subseteq AB^*
\end{aligned}
$$

□

b)   Since $A \subseteq B$, we can rewrite B as $A \cup \overline{A}$ where $\overline{A} = \{b \mid b \in B, b \notin A\}$

$$
\begin{aligned}
B^* &= (A \cup \overline{A})^* \\
&= A^* \cap \overline{A}^* \cap B_1, \qquad B_1 = \{(B^* \, a \, B^* \, a_1 \, B^*) \vee (B^* \, a_1 \, B^* \, a \, B^*) \mid a \in A, a_1 \in \overline{A}\} \\
&\Rightarrow A^* \subseteq B^*
\end{aligned}
$$

□