MA0301 Exercise 11

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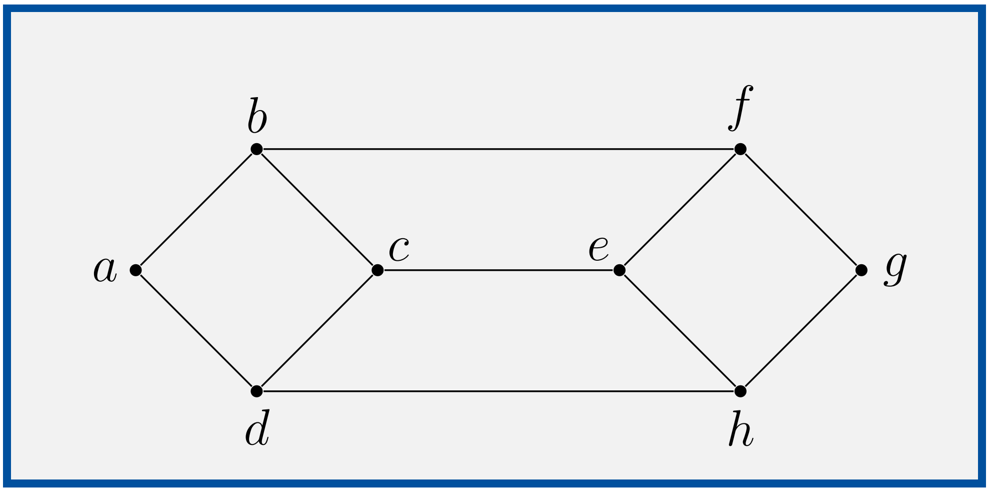
1

$$n^{n-2} = 4^{4-2} = 4^2 = 16$$

2

To solve this exercise, I chose to implement the algorithm in python

In order to keep track of the nodes, I have given them the following labels



scripts/Kruskal.py

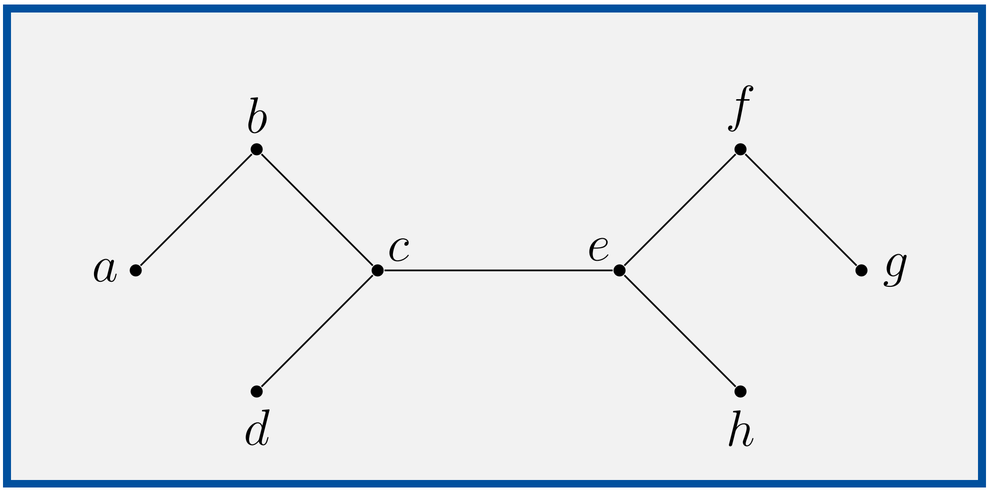
```python
def kruskal(vs, es):
  f = []
  sets = [set(v) for v in vs]

  find_set = lambda v: [x for x in sets if v in x][0]

  def merge_sets_that_contains(u, v):
    setv = find_set(v)
    newsets = [x for x in sets if v not in x]
    newsets = [setv.union(x) if u in x else x for x in newsets]
    return newsets

  sorted_es = [e for e,w in sorted(es, key=lambda e: e[1])]

  for (u, v) in sorted_es:
    if find_set(u) != find_set(v):
      f += [(u, v)]
      sets = merge_sets_that_contains(u,v)

  return f

if __name__ == '__main__':
  vs = [chr(i) for i in range(ord('a'), ord('h') + 1)]
  es = [
    (('a', 'b'), 1),
    (('b', 'c'), 5),
    (('c', 'd'), 3),
    (('d', 'a'), 7),

    (('e', 'f'), 2),
    (('f', 'g'), 6),
    (('g', 'h'), 8),
    (('h', 'e'), 4),

    (('b', 'f'), 10),
    (('c', 'e'), 9),
    (('d', 'h'), 11)
  ]

  print(kruskal(vs,es))
```

Output:

```
[('a', 'b'), ('e', 'f'), ('c', 'd'), ('h', 'e'), ('b', 'c'), ('f', 'g'),
```

```
('c', 'e')]
```

When we connect the nodes, we get the minimal spanning tree:



$\boxed{3}$

a) By counting the vertices, edges and regions, we can see that

$$|V| = 17$$
$$|E| = 34$$
$$|R| = 19$$

By applying Eulers theorem, we can confirm that this is a possible graph

$$\begin{aligned} V + R - E &= 2 \\ 17 + 19 - 34 &= 2 \\ 36 - 34 &= 2 \\ 2 &= 2 \end{aligned}$$

b) By counting the vertices, edges and regions, we can see that

$$|V| = 10$$
$$|E| = 24$$
$$|R| = 16$$

By applying Eulers theorem, we can confirm that this is a possible graph

$$\begin{aligned} V + R - E &= 2 \\ 10 + 16 - 24 &= 2 \\ 26 - 24 &= 2 \\ 2 &= 2 \end{aligned}$$

$\boxed{4}$

Every edge touches 2 regions. And every is connected to at least 5 edges. Therefore the amount of edges will be

$$E \geq \frac{53 \cdot 5}{2} = 132.5$$

Since the amount of edges has to be an integer, we can round it up to $E \geq 133$

Now we can use Eulers theorem for planar graphs to determine the amount of vertices

$$\begin{aligned} V + R - E = 2 \\ V &= 2 - R + E \\ V &\geq 2 - 53 + 133 \\ V &\geq 82 \end{aligned}$$

Therefore $|V| \geq 82$

$\boxed{5}$

a)

By flipping the matrix once vertically and once horizontally, the matrix will equal the other matrix.

Because flipping a matrix is a bijective function, composing two of them will also make a bijective function.

After checking that the last matrix is a valid undirected graph, it is safe to conclude that the graphs are isomorphic

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cong \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

b) By the same reasoning as **a)**, we have the following

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \cong \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$\boxed{6}$

a)

$$uv = ababbab$$
$$|uv| = 7$$

b)

$$vu = bababab$$
$$|vu| = 7$$

c)

$$v^2 = babbab$$
$$|v^2| = 6$$

$\boxed{7}$

a)

$$KL = \{ab^2, abb^2, a^2b^2, aaba, ababa, a^2aba\}$$

b)

$$LL = \{b^2b^2, b^2aba, abab^2, abaaba\}$$

$\boxed{8}$

a)

$$L^* = \{b^2\}^*$$

b)

$$L^* = \{a, b\}^*$$

c)

$$L^* = \{a, b, c^3\}^*$$

$\boxed{9}$

a) w does not belong to r because w is does neither fit a^* nor $(b \vee c)^*$

b) w does belong to r because w is exactly $(a \cdot 1)(b \vee c \cdot 2)$