MA0301 Exercise 1

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1

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Looking at the truthtable, we can see that $p \Rightarrow q$ only is false when p is true and q is false.

a)

$$\begin{aligned} p \land q &\equiv T \land F \\ &\equiv F \end{aligned}$$

b)

$$\begin{aligned} \neg p \lor q &\equiv \neg T \lor F \\ &\equiv F \lor F \\ &\equiv F \end{aligned}$$

c)

$$\begin{aligned} q \Rightarrow p &\equiv F \Rightarrow T \\ &\equiv \neg F \lor T \\ &\equiv T \lor T \\ &\equiv T \end{aligned}$$

d)

$$\begin{aligned} \neg q \Rightarrow \neg p &\equiv \neg F \Rightarrow \neg T \\ &\equiv T \Rightarrow F \\ &\equiv \neg T \lor F \\ &\equiv F \lor F \\ &\equiv F \end{aligned}$$

2

a) If triangle ABC is equilateral then triangle ABC is isosceles.

b) If triangle ABC is not isosceles then triangle ABC is not equilateral.

c) Triangle ABC is equilateral if and only if triangle ABC is equiangular.

d) Triangle ABC is isosceles and triangle ABC is not equilateral.

e) If triangle ABC is equiangular then triangle ABC is isosceles.

3

a)

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$	$\neg(p \wedge \neg q) \Rightarrow p$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

b)

p	q	r	$q \Rightarrow r$	$p \Rightarrow (q \Rightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

4

a)

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$q \Leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	F

$q \Leftrightarrow (\neg p \lor \neg q)$ is not a tautology.

b)

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	$(p \Rightarrow q) \land (q \Rightarrow r)$	$[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$[(p \Rightarrow q) \land (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ is a tautology.

$\boxed{5}$

I start by simplifying the expression, inserting q as T

$$\begin{aligned}
(q \Rightarrow [(\neg p \lor r) \land \neg s]) \land [\neg s \Rightarrow (\neg r \land q)] &\equiv (T \Rightarrow [(\neg p \lor r) \land \neg s]) \land [\neg s \Rightarrow (\neg r \land T)] \\
&\equiv (\neg T \lor [(\neg p \lor r) \land \neg s]) \land [\neg s \Rightarrow \neg r] \\
&\equiv (F \lor [(\neg p \lor r) \land \neg s]) \land [\neg s \Rightarrow \neg r] \\
&\equiv [(\neg p \lor r) \land \neg s] \land [\neg s \Rightarrow \neg r]
\end{aligned}$$

p	r	s	$\neg p$	$\neg r$	$\neg s$	$\neg p \lor r$	$(\neg p \lor r) \land \neg s$	$\neg s \Rightarrow \neg r$	$[(\neg p \lor r) \land \neg s] \land [\neg s \Rightarrow \neg r]$
T	T	T	F	F	F	T	F	T	F
T	T	F	F	F	T	T	T	F	F
T	F	T	F	T	F	F	F	T	F
T	F	F	F	T	T	F	F	T	F
F	T	T	T	F	F	T	F	T	F
F	T	F	T	F	T	T	T	F	F
F	F	T	T	T	F	T	F	T	F
F	F	F	T	T	T	T	T	T	T

The statement is only true when p, r and s are false.

$\boxed{6}$

a)

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$p \Rightarrow q$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

b)

p	q	r	$q \vee r$	$p \Rightarrow (q \vee r)$	$\neg r$	$p \Rightarrow q$	$\neg r \Rightarrow (p \Rightarrow q)$
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	F	F	T
T	F	F	F	F	T	F	F
F	T	T	T	T	F	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	T	T	T

$\boxed{7}$

a)

$$\begin{aligned}\neg((p \wedge q) \Rightarrow r) &\equiv \neg(\neg(p \wedge q) \vee r) \\ &\equiv \neg\neg(p \wedge q) \wedge \neg r \\ &\equiv (p \wedge q) \wedge \neg r \\ &\equiv p \wedge q \wedge \neg r\end{aligned}$$

b)

$$\begin{aligned}\neg(p \Rightarrow (\neg q \wedge r)) &\equiv \neg(\neg p \vee (\neg q \wedge r)) \\ &\equiv \neg\neg p \wedge \neg(\neg q \wedge r) \\ &\equiv p \wedge (\neg\neg q \vee \neg r) \\ &\equiv p \wedge (q \vee \neg r)\end{aligned}$$

$\boxed{8}$

α	β	γ	$\alpha \vee \beta$	$\beta \vee \gamma$	$(\alpha \vee \beta) \vee \gamma$	$\alpha \vee (\beta \vee \gamma)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

$\boxed{9}$

a)

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$p \Rightarrow (p \vee q)$ is a tautology

b) Because $\neg(p \Rightarrow (p \vee q))$ is the negation of $p \Rightarrow (p \vee q)$, which we have already evaluated to be a tautology, this has to be a contradiction and thus unsatisfiable.

c)

p	q	$p \Rightarrow q$	$p \Rightarrow (p \Rightarrow q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$p \Rightarrow (p \Rightarrow q)$ is satisfiable.

$\boxed{10}$

a) $\neg p \Rightarrow (q \Leftrightarrow r)$

b) $r \Rightarrow \neg p$

c) $\neg r \wedge (p \wedge q)$

d) $p \Rightarrow (r \wedge q)$

e) $\neg q \wedge r$